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Remarks Concerning the Determination of Electron Distribution in the Ionosphere by Means of Radio Soundings

IRVIN KAY

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REMARKS CONCERNING THE DETERMINATION OF ELECTRON DISTRIBUTION IN THE IONOSPHERE BY MEANS OF RADIO SOUNDINGS*

Irvin Kay

Morris Kline
Director

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For a number of years the pulse sounding method of Breit and Tuve[1] has been one of the most important tools for investigating the structure of the ionosphere. In this method a pulse modulated electromagnetic wave with an appropriate carrier frequency is transmitted vertically from the ground toward the upper atmosphere, and the time delay of the reflected pulse as a function of the carrier frequency is measured. The analysis of data obtained by the method of Breit and Tuve has provided much of the conceptual picture of the ionosphere which investigators have formed. However, from time to time some doubt has been expressed in the literature about the validity of certain assumptions used to relate pulse sounding data to the ionosphere's structure. In fact, it appears likely that in many cases the mathematical analysis ordinarily applied is too crude even to justify certain conclusions which are almost qualitative in nature such as the number of distinct layers present or the heights of maximum electron densities during a given experiment. Some indication of the pitfalls which may occur in the usual analysis, as well as the possibility of improving the results will be discussed here.

This discussion will be limited to what is probably the simplest model of electromagnetic wave propagation in the ionosphere for which the free electron density is assumed to vary with height. It will be assumed that the variation is strictly one-dimensional so that the ionized medium is plane stratified. In addition, the earth's magnetic field and losses in the propagating wave due to collisions of the free electrons will be neglected. Under these assumptions the effective index of refraction of the medium has the form[2], [3]

$$(1) \qquad n(x,k) = (1 - V(x)/k^2)^{1/2} ,$$

where $V(x)$ is a function of the height x and is proportional to the free electron density while k is the wave number in free space:

$$k = \omega/c \tag{2}$$

in terms of the circular frequency ω and the free space velocity of light c. The field component $u(x,k)$ of a c.w. propagating in the ionized medium satisfies the differential equation

$$d^2u/dx^2 + \left\{k^2 - V(x)\right\} u = 0 \, . \tag{3}$$

A standard method for analyzing the time delay data obtained from a pulse sounding is based on the fact that the time delay $\tau(k)$ can be expressed roughly by means of the relation

$$\tau(k) = 2 \int_0^{x_h} dx/v_g \, , \tag{4}$$

where v_g is the group velocity of the pulse and x_h is the so-called reflection height, at which the index of refraction vanishes. Usually this expression is given in terms of the virtual height $h'(k)$, defined to be half the time delay multiplied by the free space velocity of light. In terms of the virtual height there is a relation equivalent to (4):

$$h'(k) = k \int_0^{x_h} \left\{k^2 - V(x)\right\}^{-1/2} dx, \tag{5}$$

where x_h is the value of x at which $V(x)$ is equal to k^2. After $h'(k)$ is given by the experimental data, the integral equation (5) can be solved by a standard method for the function $x(V)$, called the true height [4]. The

desired function V(x) describing the electron density distribution is effectively determined, of course, by the true height x(V).

The time delay $\tau(k)$ is only approximately defined from the experimental point of view because in a sounding when the reflected pulse returns it is distorted by the dispersive effects inherent in the index of refraction n(x,k). It is possible, however, to define the time delay precisely[5]. For this purpose consider the complex reflection coefficient r(k) at the ground corresponding to waves propagating according to the differential equation.(3). If the phase of r(k) is $\theta(k)$ so that

$$r(k) = |r(k)| \exp\left\{i\theta(k)\right\},$$

then the virtual height is given by

$$h'(k) = d\theta/dk. \tag{6}$$

The time delay and thus the virtual height can also be defined equivalently in terms of the pulse, even when the dispersion is very pronounced[6]. The pulse time delay $\tau(k)$ is the time delay of the centroid with respect to time of the reflected instantaneous power relative to the centroid of the instantaneous power of the incident pulse used in the sounding, provided the incident pulse is very long.

Another source of difficulty occurs in the method associated with the integral equation (5) in that the standard solution can be obtained only when V(x) is a monotonically increasing function of x. Thus, the method applies only to a height in the ionosphere up to which the electron density is always increasing. In particular, no accurate information can be deduced about the various layers which may occur nor even about the maximum electron density of a single layer, for a breakdown in the approximation will generally take place somewhat before the height of maximum density is reached.

In principle, the reconstruction of V(x) can be carried out exactly from a complete knowledge of the reflection coefficient r(k) at all frequencies [7]. One can hardly expect this much data to be available, however. Hence there arises the question of just how much information about V(x) can be squeezed out of the data which is available. Unfortunately, at the present time it seems extremely difficult to find a complete answer to this question. Some idea can be obtained about what cannot be asserted, though, by considering special cases. It can be seen, for example, that there is not necessarily a one-to-one correspondence between the local maxima of the virtual height versus frequency curve and the layers occurring in the ionized medium. In fact, examples exist with a single maximum in the virtual height versus frequency curve corresponding to an electron density distribution function V(x) having an infinite number of local maxima.

At frequencies well below the ultimate critical frequency of the ionosphere the virtual height provides essentially all the information it is possible to obtain about the reflection coefficient, whose modulus will then be practically one. Consider now the case where there are two layers at the bottom of the ionized medium such that the electron density of the higher layer is much greater than that of the lower. Let $|r_1|$ be the modulus and θ_{r_1} the phase of the reflection coefficient of the lower layer by itself. Let $|r_2|$ be the modulus and θ_{r_2} the phase of the reflection coefficient of the upper layer by itself. Let θ_{t_1} be the phase of the transmission coefficient of the lower layer by itself. The reflection coefficient of the combination of the two layers is

$$(7) \quad r = \left\{ |r_1| \exp(i\,\theta_{r_1}) + |r_2| \exp\left[i(\theta_{r_2} + 2\,\theta_{t_1}) \right] \right\} \times \left\{ 1 + |r_1||r_2| \exp\left[i(2\theta_{t_1} + \theta_{r_2} - \theta_{r_1}) \right] \right\}^{-1} .$$

If $|r_2| = 1$, (7) takes the form

$$(8) \quad r = \left\{\exp(i\phi)\right\}\left\{\exp\left[i(2\theta_{t_1} + \theta_{r_2})\right]\right\},$$

where ϕ is the phase of the complex number of unit modulus:

$$(9) \quad \exp(i\phi) = \left\{|r_1| \exp\left[i(\theta_{r_1} - 2\theta_{t_1} - \theta_{r_2})\right]+1\right\}\left\{|r_1|\exp\left[i(2\theta_{t_1} + \theta_{r_2} - \theta_{r_1})\right]+1\right\}^{-1}.$$

If the frequency is well above the critical frequency of the lower layer but below that of the upper layer, $|r_1|$ will be very small while $|r_2|$ will still be nearly one. In this case

$$(10) \qquad r \sim \exp\left[i(2\theta_{t_1} + \theta_{r_2})\right].$$

One may interpret the derivative with respect to frequency of the phase of a transmission coefficient according to the physical idea of the pulse group velocity. The phase derivative is the increment of transit time contributed by the medium associated with the given transmission coefficient. It follows from (10) that in order to find the virtual height appropriate to the upper layer by itself after a measurement is made of the time delay of a pulse reflected from the combined layers one must correct this measurement by subtracting twice the increment of transit time of the pulse through the lower layer. This simple correction only works, of course, when the frequency is far enough above the critical frequency of the lower layer so that the quantity $|r_1|$ is nearly zero. Otherwise a further correction must be made involving the quantity ϕ in (8) and (9).

A.L. Manning[8] has pointed out the need for correcting the virtual height measurement; moreover he has shown that the result is not unique when the standard analysis based on the integral equation (5) is used to obtain the

information about the lower layer needed for the correction. This is due to the fact that one cannot determine from the usual analysis the nature of the lower layer beyond the height of its maximum electron density.

From the fact that

$$\theta \sim \lim_{x\to\infty} \left\{ \int_0^x \left[k^2 - V(x)\right]^{1/2} dx - kx \right\}, \tag{11}$$

where θ is the phase of the transmission coefficient of an ionized medium which begins at $x = 0$, it can be seen in general that the corresponding transit time increment is approximately

$$\Delta\tau = \int_0^\infty V(x)\,dx/2k^2c, \tag{12}$$

for frequencies well above the greatest critical frequency of the medium. It follows from (11) and (12) that the correction indicated in expression (10) can be obtained from a knowledge of the total number of free electrons in the lower layer.

Clearly, the problem of obtaining the total number of free electrons in the lower layer must be approached by methods other than the standard one based on the integral equation (5). One possibility that may lead to the desired result depends on the approximation of a reflection coefficient by a rational function of the wave number k. If such an approximation can be made, a simple expression for the transmission coefficient can be obtained. For example, if the reflection coefficient r(k) is a rational function of k with simple poles λ_ν, $\nu = 1,\ldots,n$ in the lower half of the complex k plane, and if $\mathcal{K}_\nu$, $\nu=1,\ldots,n$ are the roots in the upper half of the complex k plane of the equation

(13) $1 - r(k)r(-k) = 0,$

then the corresponding transmission coefficient $t(k)$ is given by

(14) $$t(k) = \prod_{\nu=1}^{n} \left\{(k+\kappa_\nu)/(k-\lambda_\nu)\right\}.$$ [9]

The best approximation of $r(k)$ by a rational function of k evidently must be an approximation based on the virtual height, i.e., the phase of $r(k)$, in a range of frequencies which does not have to extend too far above the critical frequency of the lower layer.

It should be mentioned that a closed form expression for $V(x)$ can be obtained when $r(k)$ is a rational function of k, although the expression may be quite complicated in some cases.[9] More generally, the problem of obtaining $V(x)$ from an arbitrary reflection coefficient can be reduced to solving a Fredholm integral equation[10]. To carry out the determination of $V(x)$, first define

(15) $$R(x+t) = \frac{1}{2\pi}\int_{\infty}^{\infty} \left\{r(k)\exp\left[-ik(x+t)\right]\right\} dk .$$

Next solve the integral equation

(16) $$\int_{-\infty}^{x} R(t+y)K(x,y)dy + K(x,t) + R(x+t) = 0, \quad t \leq x$$

for the function $K(x,t)$ for every x and all values of t in the range $-\infty < t \leq x$. The function $V(x)$ is then given by the equation

(17) $$V(x) = 2\, dK(x,x)/dx.$$

The Fourier transform of (15) in many cases can be evaluated by the method of stationary phase. If this is done, the kernel $R(x+t)$ of (16) will depend on the virtual height and its derivatives with respect to the wave number k. In principle, then, one can obtain the function $V(x)$ from the virtual height, whether or not the electron density is a monotonic function of height.

If a method such as this one is to be used properly to analyze the free electron distribution of an ionized gas, it is probably necessary to improve the measurement technique for obtaining virtual height data. To some extent this can be done by using a test pulse which is as long as possible rather than a short one which is now customarily used in pulse sounding experiments [6]. A further check on the data might be provided by measuring the amount of the dispersion of the reflected pulse at each frequency, particularly in the neighborhoods of the critical frequencies where the spread due to dispersion is sometimes quite large. Since the dispersive distortion of the reflected pulse is an effect which depends entirely on the reflection coefficient, a measurement of this distortion should provide additional information about the reflection coefficient.

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